Tilburg University

The Dynamics of a General Purpose Technology in a Research and Assimilation Model

Nahuis, R.

Publication date:
1998

Link to publication in Tilburg University Research Portal

Citation for published version (APA):
Nahuis, R. (1998). *The Dynamics of a General Purpose Technology in a Research and Assimilation Model*. (CentER Discussion Paper; Vol. 1998-119). Macroeconomics.

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Download date: 05. May. 2023

The Dynamics of a General Purpose Technology in a Research and Assimilation model*

Richard Nahuis

Tilburg University and CentER

P.O. Box 90153

5000 LE Tilburg

r.nahuis@kub.nl

Abstract

Where is the productivity growth from the IT revolution? Why did the skill premium rise sharply in the early eighties? Were these phenomena related? This paper examines these questions in a general equilibrium model of growth. Technological progress in firms is driven by research aimed at improving the production technology and by assimilation of ideas or principles present outside the firm. A new general purpose technology like the IT revolution generates an initial slowdown in economic growth and an increase in inequality.

JEL Classification: O31, O33, O41, J31, F43

Keywords: R&D spillovers, productivity growth, general purpose technology, wage inequality

First Draft: January 1998

Revised: October 1998.

* The author is indebted to Theo van de Klundert for suggestions and encouragement. Suggestions and support by Jan Boone, Bas Jacobs, Patrick Francois, Henri de Groot, Sjak Smulders and Harald Uhlig have contributed to the paper. Lex Meijdam is greatfully acknowledged for providing the shooting package.

The Dynamics of a General Purpose Technology in a Research and Assimilation model

by Richard Nahuis

Where is the productivity growth from the IT revolution? Why did the skill premium rise sharply in the early eighties? Were these phenomena related? This paper examines these questions in a general equilibrium model of growth. Technological progress in firms is driven by research aimed at improving the production technology and by assimilation of ideas or principles present outside the firm. A new general purpose technology like the IT revolution generates an initial slowdown in economic growth and an increase in inequality.

1. Introduction

Computers are now used in the production process of virtually every good or service. Moreover, numerous new goods and services have been made possible by the advances in computer technology. The pervasiveness of computers makes one wonder whether the invention of the semi-conductor, the heart of computer technology, did mark the beginning of a new industrial revolution. However, despite the omnipresence of information technology (IT[1]), the "revolution" does not seem to have brought a revolution in productivity development. Or, to summarise the so-called productivity paradox: "You can see the computer age everywhere but in the productivity statistics" (Solow, 1987).

Did the IT revolution lead to increased inequality? The massive introduction of computers did not bring spectacular productivity gains, however, it seems to have lead to increased inequality on the labour market. The increase in inequality between skilled and unskilled workers is an undisputed empirical phenomenon. To explain this

[1]Computer and information technology are used in the paper interchangebly to indicate information processing equipment, software and applications of microprocessors in general.

phenomenon the three main suspects are, education, trade with low-wage countries, and technological change. Educational attainment has increased, so that explanation seems to go in the wrong direction. Trade does not seem to account for a large part of increased inequality. So technology is left to explain the remainder. The obvious candidates causing the factor bias in technology are computers and IT.[2] So computers seem to have increased inequality without paying off in terms of productivity. Is there a relation between these observations?

The story this paper tells is simple. The computer is not simply a new gadget that is installed and improves productivity. The computer opens new opportunities with respect to the organisation of work and the innovative process. The full benefits of the IT revolution are not realised immediately. It takes time and resources to see and learn about the possibilities the new technology offers. Up to now the computer still imitates a paper-oriented culture and discoveries of new opportunities are still being made. Implementation of a technology with characteristics as outlined above, such a technology is called a General Purpose Technology (GPT), takes place throughout the economy and typically generates a cycle. For analysing such a learning process it is usefull to think about the process driving technological progress in a more subtle way than the one dimensional perspective that usually suffices.[3] Imagine a firm where a part of the workforce has tasks with explicit learning possibilities and hence these workers can generate improvements in production technology.[4] What is going to happen when a technology like the semi-conductor / computer arrives? First, the R&D workers are going to assimilate the opportunities the technology offers, that is for example office clerks are going to play with e-mail and the internet to find out what is in it for them and car manufacturers are going to explore the computer technology. Once the R&D

[2]See Krueger (1993) for evidence on the wage premium due to computer use. For a more thorough and extensive discussion of the potential explanations for the increase in wage inequality, see Nahuis (1997).

[3]In most endogenous growth models a single R&D activity generates blueprints, see for example Grossman and Helpman (1991).

[4]These workers are called in the remainder R&D workers but you might want to think of these in a more broad sense as all workers who experience substantial learning possibilities in their jobs, including for example most white-collar workers.

workers assimilated the opportunities the new technology offers this is going to pay-off in firm-specific applications of the GPT, that is, e-mail turns out to be an efficient way of commucating and the internet appears to be a productivity enhancing source of information for office clerks, and car producers find it usefull to develop chips to make cars more reliable. So what goes on is that a GPT distracts R&D workers from direct productive R&D, this causes growth to slow down. But, once the opportunities of the GPT are recognized, research is more effective and growth accelerates. Assume moreover that skilled workers have an advantage in research, learning and assimilation and the introduction of computers improves the relative wage of skilled workers too (Bartel and Lichtenberg, 1987). Hence the two observations are related!

The purpose of this paper is to show that in a more realistic formulation of the learning process the introduction of a GPT generates a slowdown in productivity growth without (large) fluctuations in R&D labour, as these are empirically not observed. Moreover the paper shows that sluggisch productivity growth and increased wage inequality might be related phenomena. Actually the paper shows that the two observations are the consequence of one and the same thing: a GPT.

The paper proceeds as follows. The next section discusses the literature on GPTs and growth cycles. In section 3 the learning process outlined above is embedded in a model with skilled and unskilled workers. Section 4 analyses the steady state properties of the model. Section 5 analyses the long run impact of a GPT, whereas in section 6 the impact effects of a GPT are analysed numerically with a calibrated version of the model. Section 7 concludes.

2. General Purpose Technologies

Economic growth's most important driving force is technological progress. Technological progress in turn seems to be driven by a few major technical or organisational breakthroughs. Examples are: the concept of a factory as a way of organizing work, the steam engine, electricity and the transistor. These concepts, mostly breakthroughs in engineering, have turned out to be widely applicable throughout the economy. Application of such a generic function in a specific context requires investment. Most of the applications of the, what later turned out to be, general concepts have never come to the minds of the original inventors, but slowly possibilities were recognized and also the GPTs themselves improved over time. Some common features of GPTs can be distilled from the rich descriptive material provided by David (1990 and 1991). David compares the general purpose engine of the previous *fin de siecle*, the electrical engine, with the current one, the information technology, and finds remarkable parallels. Analogous are the sluggish labour productivity growth at the turn of the century, constancy of real wages, and a new technology that was introduced everywhere but did not contribute to recorded productivity growth. As the macro-economic tendencies at the end of the previous century seem similar to today's, we might learn from understanding why no productivity gains from early electrification were recorded.[5] First, it took some 20 to 30 years before adoption of electrical engines was substantial. Second, early adopters used electricity driven systems that were backed up by mechanical power derived from steam or water.[6] The third explanation is crucial for the aspects of technological change we focus on. It turned out that substantive productivity gains of electrification were only accomplished once it was recognized that factories could be designed in a previously unthinkable way and hence work could be organised much more efficiently. Illustrative evidence for this claim is that early applications of electrical engines were used to lift water back up to the top of the water-wheel while keeping the factory organised by the restriction that all machines needed to be connected by belts to the single power source. David (1989, p.23) argues that "The advantages of the unit drive [that is for every machine a separate power source, RN] for factory design were manifold, extending well beyond the savings in inputs of fuel...". Hence, an appropriate way of looking at a new GPT might be that it fuels innovation.[7]

---

[5]Also at that time, as today, measurement problems might have played a role. The replacement of gas lightning by electrical lightning improved brightness, safety etc. without directly affecting recorded productivity.

[6]Hence, if anything, recorded labour productivity might have increase, not TFP, as the capital labour ratio increased due to a "double" capital stock.

[7]Bresnahan and Trajtenberg (1995) cite Griliches' study of hybrid corn. Hybrid corn is a technology that generated completely new possibilities in the field of agriculture: "Hybrid corn was the invention

The key ingredients of our model, that copes with the diffusion process of GPTs, as described by David (1991) and Bresnahan and Trajtenberg (1995), are the following. The first characteristic of a GPT is that it is potentially beneficial to all firms. In the model a GPT generates new possibilities for innovation for all firms. The second characteristic of a GPT is that firms have to invest in assimilating the nature of the new GPT before they can put effort in developing useful applications of the GPT. Hence we model assimilation of new ideas out of a pool of knowledge (including a GPT and spillovers of other firms' research activity) that yields a stock of accumulation capabilities. This stock is a necessary input for research for directly applicable knowledge. For reseach for directly applicable knowledge to remain a viable activity the stock of accumulation capabilities should grow. For an earlier application of the two-stage research structure, see Rustichini and Schmitz (1991).[8] Finally, fruitfull application of the GPT by a specific firm generates spillovers that may interact with the GPT. This indirecly implies that an inherent potential for improvements of the GPT exists (that is the third characteristic David attributes to GPTs). We examine cases with and without a positive interaction effect. Finally, the model distinguishes two types of workers; skilled workers fully specialized in research and unskilled workers that are only suitable for production.

2.1 Related Literature

The dynamics of a GPT have recently been analysed formally by Helpman and Trajtenberg (1994, further HT). They develop a general equilibrium model where GPTs require complementary inputs before they can be applied profitable in the production process. Complementary inputs developed for a previous GPT are not suited for use with a newly arrived GPT. The invention of complementary inputs requires a fixed labour input. The arrival of subsequent GPTs causes cycles. A typical cycle consists of two phases, a phase where firms produce final goods with the old GPT and components are being developed for the new GPT, and a second phase where final goods producers

of a method of inventing, a method of breeding superior corn..." (P.501, 1957).

[8] Rustichini and Schimtz (1991) use such a structure to analyse optimal technology policy. The structure of this learning process resembles the idea of learning to learn, see Stiglitz (1987).

switch to the new GPT and the development of components for that GPT is continued. A consequence of such a technology cycle is that GDP declines in the first phase as workers switch from production to research to invent new inputs and increases again in the second phase once the new technology is implemented.[9] [10]

Our analyses is in the spirit of the work by HT but deviates in the mechanisms driving the application of a new GPT and hence differs importantly in the empirical implications. The mechanism this paper introduces is new in the literature on GPTs and growth cycles. The first difference with respect to the mechanism is that in the set up of our model existing firms have to cope with the new GPT instead of new firms as in HT. In this sense our analysis is complementary to HT's.

The second difference is that we analyse the process of implementation instead of the decision to adopt. In vintage-type models the decision to adopt is analysed from various perspectives that have strong similarities. For example, adoption of technology in vintage-capital models generates growth cycles due to the assumption that the starting level of expertise in using a technology after adoption depends negatively on the pace of technological progress. The IT revolutions is seen as a positive productivity growth shock in investment specific technology, see Greenwood and Yorukoglu (1997) and Yorukoglu (1998).[11] Helpman and Rangel (1998) provide a different perspective by analysing the decision of workers to adopt a new technology. Workers are heterogeneous with respect to the experience they have with a previously dominant technology. Similar to vintage-capital models a slump occurs if the efficiciency of workers who start using the new technology is lower than with previous technology and

[9] An extension of the model where skilled workers are specialized in research and unskilled workers in production allows analysis of the skill premuim over the cycle. In the first phase, the skill premium increases, in the second phase a non-monotonic pattern prevents derivation of unambiguous results. Unattractive features of this version of the model are the decline of real wages of skilled workers in the second phase and the fact that there is no allocation decision what so ever; skilled workers produce new components and unskilled workers produce final goods.

[10] Eriksson and Lindh (1997) endogenize the arrival rate of GPTs and allow for intertemporal spillovers in the HT framework. Helpman and Trajtenberg (1996) analyse the diffusion of GPTs throughout the economy over heterogenous final goods.

[11] The IT revolution thus induces an increased renewal of plants. These plants all need to master the technology, yielding a temporary slowdown in growth and an increased skill premium as implementation and learning are skill intensive.

a slump is more likely when a larger proportion of the workforce switches (the latter is positively related to the learning speed with the new technology). Again slightly different, heterogeniety in firm productivity combined with learning by doing in the capital goods producing sector (instead of workers using the technology) generates possibly a slump (Felli and Ortalo-Magné, 1997). A slump is not necessary, however, as the most productive firms will switch immediatly and therby increase output. This counteracts the fact that the least productive firms lower investment in the old technology. If the first effect outwheighs the second the arrival of a new technology will be followed by a boom.[12]

Third, the R&D process in our model differs from the literature. Research increases productivity and the assimilation activity to comprehend new ideas is required to keep up the research potential. On the one hand this two stage learning process resembles other approaches with two different types of R&D, as R&D-labour can perform different types of research. However, our approach contrasts these appraoches in the sense that in other approaches firms have to decide either to aim at breakthroughs or to aim at improvements, as in our model these activities are complements instead of substitutes (Cheng and Dinopoulos, 1996)[13].

The (empirical) implications of our model are substantially different from the existing literature. First, in the model set forth in this paper no reallocation from R&D to production is required to generate a slump in productivity growth. The empirical implication of the HT approach is that the occurrence of cycles in growth should be accompanied by strong fluctuations in resources devoted to R&D. However:

[12]The adoption of the diesel locomotive in the US was not associated with an initial slump in output. Other factors in the model that counteract a decline in output are the price decline on the used market for capital goods and the more intense use of old capital. Both might have been relevant for the example.

[13] Cheng and Dinopoulos generate growth cycles by a R&D sector that can target either on breakthroughs or improvements. If the return to improvements does not diminish quickly a breakthrough will be followed by sequence of improvements, hence generating a cyclical pattern. See also Jovanovic and Rob (1990) on extensive and intensive search. Stein (1997) models two dimensions along which technology improves. Incumbent firms increase their lead over potential entrants in one technological dimension by learning-by-doing. Once this lead is substantial, only very favourable circumstances induce entrants to enter, who, by doing so, make future entry much more likely (and hence this mechanism generates an uneven growth pattern).

"...fluctuation in research and development and in employment of resources are not large... enough to explain significant and progressive fluctuations in output" Felli and Ortalo-Magné (1997, p.4). [14] Also Andolfatto and MacDonald (1998) need too large fluctuations in R&D labour to replicate post-WW-II data in a model where technologies that differ in quality and difficulty to acquire are innovated or assimilated. Our model shows that it is possible to generate growth cycles in a R&D model without large fluctuations, actually no fluctuations at all, in R&D labour. Secondly, our structure generates a slowdown in output growth, while introducing a superior technology, without the necessity to assume creative destruction, forgetting, or some kind of incompatibilities.

3. The model

The impact of the emergence of a new general purpose technology will be analysed in a general equilibrium model.

3.1 Description of the model

The model economy consists of a "large" number of firms, *N*, that produce a differentiated variety of a consumption good. Firms are located equally spaced on a circle. The closer they are the more similar their knowledge. The economy is populated with *H* skilled workers and *L* unskilled workers who both supply their labour fully inelastic. We will consider the case of symmetric industries. In a symmetric allocation, every firm can allocate *L/N* and *H/N* workers; denote the former L_N and the latter H_N. In the remainder it will be shown that the analysis might be expressed in terms of the representative firm and consumer as the

number of firms does not play a role. Consider a representative firm *i*, indexed $i \in \{1,...,N\}$. Firm *i* produces good *i* with a linear production technology,[15]

$$x_i = h_i L_i \ . \qquad (1)$$

[14] This point was first made by Aghion and Howitt (1996).

[15] Time subscripts are not introduced where it leads to no confusion.

Unskilled labour, L, produces good x with productivity h. Notice, for later reference, that the production technology features constant returns in the traditional production factor, labour. The stock of productive knowledge is firm-specific in the sense that only firm i is able to produce with knowledge stock h_i.[16] Firm i accumulates knowledge by investing in R&D using the following technology

$$\dot{h}_i = A_h (h_i H_{Ri})^{1-\alpha} f_i^{\alpha} \ , \ 0 < \alpha < 1 \ . \qquad (2)$$

A_h is a research productivity parameter, H_R is skilled labour devoted to research activity and f is the stock of accumulation capabilities. Several features of this specification are worth noticing. First, the technology for accumulating productive knowledge exhibits decreasing returns to h. This implies that relying fully on internally generated experience (reflected in h) is insufficient to keep growth going.[17] Hence, as the amount of labour is assumed to be fixed, to have growth that does not peter out, the second asset (accumulation capabilities) should grow. The second feature is that there are decreasing returns to labour in knowledge production. That is, unlike the production technology for goods, a replication argument does not hold here. Hence, given the amount of assets (h and f), doubling the amount of labour at a point in time does not lead to a twofold increase in the flow of new ideas or productive knowledge (*cf.* Jones, 1995).[18]

Accumulation capabilities serve as an asset in the research process for new productive knowledge. These capabilities are accumulated according to

[16]This can be motivated by assuming that the effective application of knowledge is only possible if a firm has developed knowledge by own R&D. Alternatively, patents could take care of monopolising the knowledge stock.

[17]For most readers the most nearby example, to make the discussion less abstract, is doing research while only reading own previous work. It will be clear that in the end your effective additions to knowledge stop.

[18]Again take the academic example: if you are working on a paper with three co-authors adding another four authours will not double output.

$$\dot{f}_i = A_f \left[\left(\frac{Z}{f_i} \right)^{\phi} f_i \right]^{\lambda} H_{Ai} \ , \ \phi > 0 \, , \lambda \leq 1 \ , \tag{3}$$

where A_f is the parameter governing the productivity of the assimilation technology and H_A is skilled labour engaged in assimilation. Z is the knowledge pool available to a firm, this knowledge pool is 'filled' with the latest generation of the GPT and knowledge spillovers. Spillovers are the more general principles, developed as a byproduct of research for firm-specific knowledge, that are indirectly useful for other firms. As only symmetric equilibria are considered we can, without loss of generality, assume that a single general knowledge pool exists, hence no firm index is added. The term between parenthesis, Z/f, is the 'learning potential' or 'knowledge gap': the size of the knowledge pool relative to the firm's accumulation capability. As ϕ is positive, a larger learning potential implies more effective assimilation. Consider two cases with respect to ϕ. First, if $\phi > 1$ 'fishing out' applies. The most effective ideas, or equivalently the most obvious ideas, are 'fished out' of the common knowledge pool first. In that case the more accumulation capabilities a firm has, the harder or less effective further assimilation will be; $\frac{\partial \dot{f}}{\partial f} < 0$ An alternative hypothesis is that firms learn to assimilate, reflected in $\phi < 1$. Then more accumulation capabilities imply easier assimilation; $\frac{\partial \dot{f}}{\partial f} > 0$. The specification features decreasing returns in Z and f together if λ is less than unity. One might expect that assimilating existing knowledge is an activity where doubling the amount of human input leads to doubling the output.[19] Therefore the specification for assimilation again reflects the replication argument. Figure 1 provides an overview of the activities of firms 1 to N, for firm 1 the dashed

[19] To continue the academic analogy: the output of writing summaries of books and papers is doubled when the input is doubled.

arrow indicates the spillover.

In the knowledge pool, spillovers from productive knowledge of other firms and the GPT play a role.[20] The knowledge pool with the GPT of generation *j* looks like:

$$Z(j)=Z(h_1,....h_N,Q(j)) \quad . \tag{4}$$

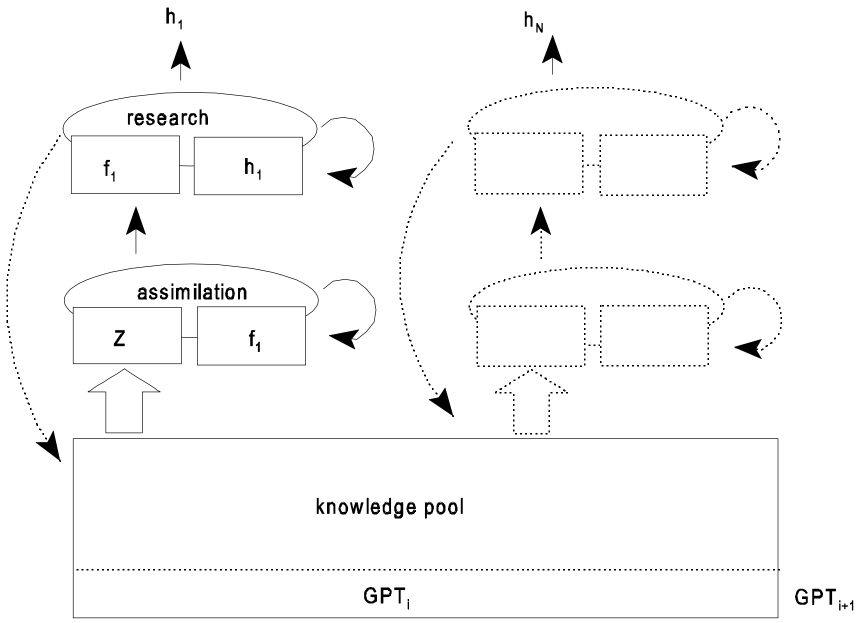

Figure 1

Note that *Z(j)* changes gradually over time due to changes in h_i for all *i*. *Q(j)* is the effective quality of the GPT of generation *j*. We assume $\partial Z/\partial h_i$ and $\partial Z/\partial Q >0$. The invention of a next generation GPT occurs serendipiditously and hence is unexpected and does not require resources. A more extensive discussion of the function *Z* is postponed until section 5.[21]

[20]The empirical literature on spillovers learns that several sources of knowledge turn out to be important in the innovative proces of firms: research by others in the economy, university research and government research. The model could be extended to allow for the latter two.

[21] The number of firms is not an explicit argument in *Z* as it is fixed throughout the analysis. Moreover, as firms are located equally spaced on a circle all firms have access to an equivalent "amount" of knowledge. An alternative motivation could be that all firms have access to the an unweighted average knowledge stock and hence N is irrelevant. De Groot and Nahuis (1998) analyse the interaction between *N* and *h* in a much simpler framework. Peretto and Smulders (1998) analyse

Table 3.1 Producer Behaviour

Producer i:

$$\max \; V_i = \int_0^{\infty} e^{-r_t} \left(x_{it} p_{it} - (H_{Rit} + H_{Ait}) w_{Ht} - L_{it} w_{Lt} \right) dt \qquad (5)$$

subject to: (1), (2), (3), (15) and $H_{Ri} \geq 0$, $H_{Ai} \geq 0$.

First order conditions (f.o.c.) are:

$$\frac{\partial \mathcal{H}}{\partial L_i} = 0 \qquad p_{xi} = \frac{\varepsilon}{\varepsilon - 1} \frac{w_L}{h_i} \qquad (6)$$

$$\frac{\partial \mathcal{H}}{\partial H_{Ri}} = 0 \qquad (1-\alpha)\, q_{hi} A_{hi} H_{Ri}^{-\alpha} h_i^{1-\alpha} f_i^{\alpha} = w_H \qquad (7)$$

$$\frac{\partial \mathcal{H}}{\partial H_{Ai}} = 0 \qquad q_{fi} A_{fi} \left[Z^{\phi} f_i^{1-\phi} \right]^{\lambda} = w_H \qquad (8)$$

$$\frac{\partial \mathcal{H}}{\partial h_i} + \dot{q}_{hi} = r q_{hi} \qquad \frac{w_L L_i}{h_i} + (1-\alpha)\, q_{hi} A_{hi} h_i^{-\alpha} H_{Ri}^{1-\alpha} f_i^{\alpha} + \dot{q}_{hi} = r q_{hi} \qquad (9)$$

$$\frac{\partial \mathcal{H}}{\partial f_i} + \dot{q}_{fi} = r q_{fi} \qquad \alpha\, q_{hi} A_{hi} (h_i H_{Ri})^{1-\alpha} f_i^{\alpha-1} + \lambda (1-\phi)\, q_{fi} A_{fi} Z^{\phi\lambda} f_i^{\lambda-1-\phi} H_{Ai} + \dot{q}_{fi} = r\, q_{fi} \qquad (10)$$

Producer behaviour is summarised in Table 3.1. Producers maximize the value function V indicating the present value of the firm, subject to the technical constraints discussed above and a downward sloping demand curve familiar to preferences with goods that are imperfect substitutes. The Hamiltonian of the formulated maximisation program is

the interaction between spillovers and changes in the number of firms.

denoted by $\mathcal{H}$. We focus on interior solutions, hence the inequality constraints are not binding (for a discussion of corner solutions is, see Appendix C).

F.o.c. (6), derived from the optimal use of unskilled labour or the optimal supply of output, shows that firms set a mark-up over the unit labour cost. The mark-up is inversely related to the price elasticity, ε. Optimal allocation of skilled workers in both the research and the assimilation activity requires the marginal cost, w_H, to equal marginal return (the lhs of equations (7) and (8)). The marginal return of productive knowledge is the marginal increase in the productive knowledge stock, valued with the shadow price, q_h. The lhs of (8) shows the marginal product of skilled workers in assimilation; the shadow price q_f times the marginal addition of accumulation capabilities. The marginal return of assimilation is increasing in Z and increasing (decreasing) in f if ϕ smaller (larger) than unity. The no-arbitrage condition (9) says that the return of investing q_h in the financial market should equal the return of investing in productive knowledge. The latter consists of three parts. The first part on the lhs of equation (9) is the direct benefit of a marginal increase in productive knowledge: the marginal increase in the value of production, $\partial x p_x / \partial h$.[22] The second term is the increase in the knowledge base, again valued at the shadow price. The third term is the capital gain term. The second no-arbitrage condition, (10), has a completely analogous structure. The first term on the lhs is the direct benefit of a marginal increase in accumulation capabilities, that is the increase in the value of direct knowledge production, $\partial \dot{h} q_h / \partial f$. The second term is the change in 'fishing potency'. If 'fishing out' applies (ϕ>1) this term is negative, indicating that accumulation of f today implies *cet. par.* a lower return to assimilation tomorrow. The third term is again a capital gain term.

Preferences and consumer behaviour are standard and presented in Table 3.2. Maximising the CRRA utility function subject to the wealth accumulation constraint yields the familiar Ramsey rule. θ and 1/σ denote subsequently the pure rate of time preference and the elasticity of intertemporal substitution. In the second stage of the budgeting problem, consumers decide on the division of their spending over different varieties at each point in time. A downward sloping demand schedule for each variety

---

[22] This is easily seen combining equation (1) and (15).

(equation (15)) results. Symmetry of firms, hence prices, results in a uniform distribution of spending over the differentiated goods.

Table 3.2 Consumer Behaviour[a b]

$$\max\ U=\int_0^{\infty} e^{-\theta_t} \frac{1}{1-\sigma} X_t^{1-\sigma} dt \tag{11}$$

$$\text{where } X_t=N\left(\frac{1}{N}\sum_{i=1}^{N} x_{it}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{12}$$

Yields:

$$\frac{\dot{X}}{X}=\frac{1}{\sigma}\left((r-\dot{P}_X/P_X)-\theta\right) \tag{13}$$

Where:

$$P_X=\left(\frac{1}{N}\sum_{i=1}^{N} p_{xi}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{14}$$

$$p_{xi}=P_X\left(\frac{x_i}{X}\right)^{-\frac{1}{\varepsilon}} \tag{15}$$

[a] For a discussion of the financial market and equilibrium asset holdings in this type of models see Van de Klundert and Smulders (1997).[b] In symmetric equilibrium: $P_X=p_x$. and $XP_X=nxp_x$; we normalise n to 1, hence $X=x$.

Consumer preferences, presented in Table 3.2, imply that variety as such does not play a role.[23]

In order to stress that no reallocation of workers from production to research is needed to generate growth cycles, we analyse the special case without any substitution possibilities between (skilled) research workers and (unskilled) production workers. Hence, the labour market is completely segmented. Aside from expositional ease, the segmentation can be motivated by the notion that skilled workers are primarily suitable for research and assimilation and unskilled workers for production. The qualitative results, we believe, would not be affected by relaxing this assumption. The segmentation of the labour market implies the following equilibrium condition:

$$H_N = H_R + H_A \ , \tag{16}$$

for skilled workers. And similarly for unskilled workers:

$$L_N = L \ . \tag{17}$$

Solution of the model and characterization of the steady state will be the topic of the next section.

4. The steady state

This section discusses the steady state. Hence, in this section we abstain from the emergence of a new (generation of a) GPT. Section 4.1 defines and section 4.2 solves for the steady state. The determination of relative wages and labour market equilibrium is discussed in section 4.3.

4.1 Definition of the steady state

A steady-state equilibrium is defined as a path where all variables grow at a constant, possibly different, rate and where the allocation of labour is time-invariant. It is easy to

[23]For the analysis of returns to variety see De Groot and Nahuis (1997).

show that Z should be homogeneous of degree $(\phi\lambda+1-\lambda)/\phi\lambda$ in average productive knowledge to have positive steady state growth (use (2) and (3)). If $\lambda<1$ this implies that the knowledge pool should grow at a higher rate than the knowledge generated by agents in the economy. Empirical reseach should answer the question whether such a relation is plausible. For the remainder of the analysis the specification is specialised to one with constant returns in assimilation and the knowledge pool *Z*, by setting $\lambda=1$. Hence, *Z* should have a long-run growth rate equal to that of *h*:

$$Z(j)=hZ\left(1,\frac{Q(j)}{h}\right) \quad . \tag{18}$$

Keep in mind that there is no steady growth in the GPT, hence in the long run the arguments of *Z* tend to a constant which is denoted *z* in the remainder. Finally we normalise prices to one. The balanced growth path is characterized as (using (6),(7),(8) and (9)):

$$\frac{\dot{X}}{X}=\frac{\dot{x}}{x}=\frac{\dot{h}}{h}=\frac{\dot{f}}{f}=\frac{\dot{Z}}{Z}\equiv g \quad . \tag{19}$$

Hence all variables grow at a common constant rate, denoted *g*. For prices holds that:

$$p_x=1 \;\Rightarrow\; \frac{\dot{P}_X}{P_X}=\frac{\dot{p}_x}{p_x}=0, \quad \frac{\dot{q}_h}{q_h}=\frac{\dot{q}_f}{q_f}=0, \quad \frac{\dot{w}_L}{w_L}=\frac{\dot{w}_H}{w_H}=g \quad . \tag{20}$$

4.2 Solution of the model

Some additional notation simplifies the exposition further. Define $u=H_R/H_N$ as the fraction of skilled labour doing research. Labour market equilibrium for high skilled workers can now be written as (using (2),(3))[24]:

[24]The expression can also be read as $\hat{h}=\hat{f}$, and $\hat{x}\equiv\frac{\partial x/\partial t}{x}$.

$$R=\left(\frac{A_h}{A_f}z^{-\phi}H_N^{-\alpha}\frac{u^{1-\alpha}}{1-u}\right)^{\frac{1}{\phi+\alpha}} . \qquad \text{(LAB)}$$

Where $R \equiv h/f$, that is the steady-state ratio of productive knowledge and accumulation capabilities. Multiplying both sides of the definition for R with z, that is constant in steady state, shows that R is proportional to the knowledge gap, Z/f, in the steady state.

Having defined the steady state by (19) and (20), the Ramsey-rule (13) can be written as $g=\frac{1}{\sigma}(r-\theta)$. To find the steady-state growth rate, the equilibrium rate of return on savings should be substituted in this expression. Firms equate the return to investment in productive knowledge and in accumulation capabilities. Hence, the rate of return can be found using only one no-arbitrage condition. Combining the no-arbitrage condition for accumulation capabilities (10) with the two static optimality conditions ((7),(8)) yields an offered return to capital of:

$$r=(1-\phi)A_f(zR)^{\phi}(1-u)H_N+\frac{\alpha}{1-\alpha}\frac{u}{1-u}A_h(uH_N)^{1-\alpha}(R)^{-\alpha} . \qquad (21)$$

The offered return is a weighted sum of accumulation equations (2) and (3) and should be read as the demand for capital. To derive an explicit expression for g turns out to be cumbersome, therefore we rely on a graphical approach. Rewriting (2) to (use the definition of R): $g=A_h(uH_N)^{1-\alpha}R^{-\alpha}$, and substituting this into the Ramsey-rule yields an expression for the required return on savings by consumers. Combining this with (21) yields an equation for the capital market equilibrium (CAP):

$$R=\left(\frac{H_N A_f z^{\phi}}{\theta}\left[\left(\frac{\alpha}{1-\alpha}-(1-\sigma-\phi)\right)u+(1-\sigma-\phi)\right]\right)^{-\frac{1}{\phi}} . \qquad \text{(CAP)}$$

To show that for certain parameters an equilibrium with positive growth exists, a graph in the u-R space is convenient, see Figure 2.[25] The LAB-curve is upward sloping; if u

[25] The parameter restriction is implied by $u>u^{\sharp}$. $u^{\sharp}=(\phi+(\sigma-1))/(\phi+(\sigma-1)+\alpha/(1-\alpha))$ hence if α close to unity $u^{\sharp}$ collapses to the vertical axis. If α goes to zero $u^{\sharp}$ goes to unity, hence the equilibrium u is close to one therefore growth will be driven almost completely by research.

is high, that is most skilled workers are allocated to accumulate productive knowledge, the knowledge gap is large. The CAP-curve is downward sloping. The intuition is that the equilibrium rate of return is the maximal rate of return attainable, as firms optimize. Hence if the knowledge gap is large, *R* high, the highest real return is attained by putting a lot of effort into assimilation (closing the knowledge gap). The equilibrium allocation and the ratio of productive knowledge to accumulation capabilities is found at the intersection of the CAP and LAB curve. The growth rate, finally is computed using equation (2) (use the definition of *R* and *u*).

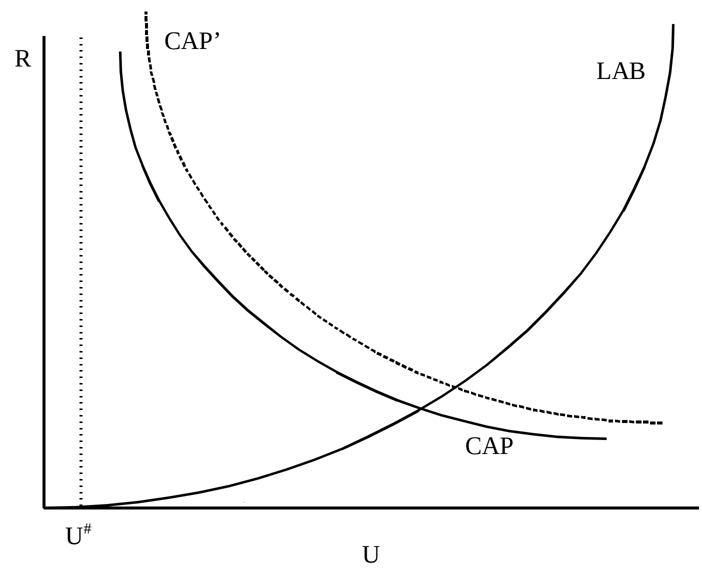

Figure 2

4.3 Relative wage determination

The model has a recursive structure in the sense that the solution for the growth rate does not require solving for the complete model. This is easily seen as *u* and *R* are determined by the intersection of the CAP and LAB curve that are both independent of the number of unskilled workers (L_N). Hence the endowment of unskilled workers and the relative wage do not affect the growth rate.[26] This section provides a digression on relative wage determination.

[26] Technically speaking, for the solution of the growth rate equation (9) is not required. After solving for *u* and *R* the relative wage can be solved for recursively.

Equating the no-arbitrage conditions (9) and (10), taking into account the definition of the steady state one can derive an expression for the relative wage ($\omega \equiv w_L/w_H$):

$$\omega = \frac{1}{1-\alpha}\frac{H_N}{L_N}u\left(\frac{\alpha}{(1-\alpha)}\frac{u}{(1-u)}+\alpha-\phi\right) , \tag{22}$$

where (7) and (8) are substituted for the shadow prices. As L_N does not affect u, ω is decreasing in the endowment of unskilled workers.

To show the working of the model's wage determination, consider the following comparative static experiment. Assume consumers become less patient, hence the rate of time preference (θ) increases. The CAP-curve shifts up (CAP' in Figure 2) and the LAB-curve is not affected. Hence, the new steady state is characterized by a larger knowledge gap and a higher fraction of the workforce doing research (u). From (22) it is easily seen that the relative wage of low skilled workers increases in u. What is the intuition for the positive relation between the relative wage of unskilled workers and the degree of impatience? Unskilled workers are specialized in final goods production. At a point in time, production possibilities are fixed as both the stock of productive knowledge and the supply of unskilled labour is fixed. Dynamically, however, the increase in relative wages is consistent with less patience. An increase in the relative wage increases the return to investment in research (see, (9)) and does not affect the return to assimilation. Therefore the return to the asset that has a direct impact on productivity is increased relative to the return to the asset that affects only future accumulation. Therefore a higher fraction of the workforce is doing research in the new steady state.

The impact of a higher rate of time preference on growth however yields an ambiguous result. Graphical inspection shows that more skilled labour is allocated towards research but with a lower productivity, as the ratio of productive knowledge to accumulation capabilities (R) is lower. By linearising the model around the steady state we can derive that an increase in the rate of time preference turns out to decrease growth unambiguously (for details and other comparative statics see appendix A and B).[27]

5. A new General Purpose Technology

This section has two purposes. First, the relation between the arrival of a new GPT and productive knowledge will be discussed. Second, the impact of an increase in the GPT's quality (Q) and the long run performance of the economy is examined.

So far the knowledge pool is defined as:

$$Z(j)=Z(h_1,....h_N,Q(j)) \quad . \tag{4}$$

The previous section established that in the long run the knowledge pool should grow with the growth rate of h to have a steady state with positive growth. At time t, GPT of quality j arrives; the arrival of a new GPT is exogenous. We analyse a one time unanticipated arrival of a GPT. We assume that at the arrival of GPT j the economy is on a stationary path, as discussed above. Furthermore, the GPT's quality is one dimensional and a new GPT is strictly better than the previous one.

The interaction of a GPT with knowledge spillovers from other firms that is available in the knowledge pool gives rise to two cases. In the first, a GPT is simply new knowledge that is widely applicable and requires time and effort to implement. In the second case, a GPT is idea generating, hence a positive interaction between spillovers and a GPT exists. In the remainder the intuition for and the long run impact of the two cases will be discussed.

In case (i) no interaction exists between the new GPT and spillovers from productive knowledge, hence Z_{hQ}=0 (where the subscript indicates the cross derivative). An example of a functional form where the cross derivative is zero is the case where h and Q enter additively in Z. So:

[27]The comparative statics results are derived under a logarithmic utility function.

$$Z(j)=\bar{h}+Q(j) \quad , \tag{4A}$$

where $\bar{h}$ is the effective spillover of firms' productive knowledge. The equivalent of this equation in words is: the new GPT is pervasive, as it affects learning possibilities in all *N* industries and it requires complementary investments to advance the performance of the technology in a specific environment. However, the technology does not positively interact with technologies available now in the future. More concrete such technologies are gadgets that turn up everywhere but whose technology does not make other technologies more productive, examples are most office supplies. What about the long run impact of such a technology? The definition of a steady state (see section 4.1) requires the ratio of productive knowledge (*h*) and accumulation capabilities (*f*) to be constant in the steady state. Both *h* and *f* grow in the steady state, hence it is easy to see that, in the long run, the improved GPT ceases to have impact on growth.[28]

In case (ii), the ideas generating GPT, there is a positive interaction of the new GPT with existing and future productive knowledge, hence $Z_{hQ}>0$. The GPT of our times, semi-conductor technology, seems to fit well in this classification. Semi-conductors do seem to improve the efficiency of all devices already developed. To infer the impact of such a GPT on long run growth we specialize *Z* to:

$$Z(j)=\bar{h}Q(j) \quad . \tag{4M}$$

In this case fruitfull application of the GPT by firms increases their productive knowledge which in turn generates spillovers that interact positively with the GPT. This mechanism implies that succesfull applications "improve" the GPT. This is the third characteristic of a the GPT David (1989) described (see section 2). The impact on long run growth is inferred from the linearised version of the model. The impact of an increase in the quality of the GPT is:

$$\tilde{g}=\Xi\phi\tilde{Q} \quad . \tag{23}$$

[28] Devide both sides of (4A) by *h*, due to (3), and notice that the impact of *Q* vanishes.

Variables with a tilde denote deviations from a steady state, hence $\tilde{x} \equiv dx/x$. Ξ is always positive. Hence, an increase in the GPT's quality leads to higher growth.[29]

6. Dynamics and calibration

The previous sections showed conditions for a new GPT to increase the long run rate of growth. The short run impact of a new GPT might however be different. This section deals with the analysis of the initial impact of a new GPT and the accompanying transitional dynamics. To that end we derive the dynamic equations of the non-linear model, calibrate the model and analyse transitional dynamics.

6.1 Dynamic equations

In section 4.2 and 4.3 reduced form equations for the model are derived in terms of the relative wage (ω), research allocation and the knowledge gap. Differential equations for these variables describe the dynamic behaviour of the economy, one equation for the state-like variable *R*, that is the ratio of two state variables (*h* and *f*), and two equations for the jump variables *u* and ω.

$$\dot{R} = R\left(A_h(uH_N)^{1-\alpha}R^{-\alpha} - A_f(zR)^{\phi}(1-u)H_n\right) , \tag{24}$$

$$\dot{u} = \frac{u}{\alpha}\left(\left(\frac{\alpha}{1-\alpha}u + \alpha(1-u)\right)H_N A_f(zR)^{\phi} - \left((1-\alpha)\omega\frac{L_N}{H_N}\frac{1}{u} + \phi\right)A_h(uH_N)^{1-\alpha}R^{-\alpha}\right) , \tag{25}$$

$$\dot{\omega} = \omega\left((1-\phi-\sigma)A_h(uH_n)^{1-\alpha}R^{-\alpha} - \theta + \frac{\alpha}{1-\alpha}(uH_N)A_f(zR)^{\phi}\right) . \tag{26}$$

The dynamic equation for the knowledge gap is easily found by log differentiating the definition of *R* and substituting the accumulation equations for productive knowledge and accumulation capabilities. The second dynamic equation is found by log

[29] The derivation of Ξ is explained in appendix B.

differentiation of the conditions for the optimal static allocation of skilled workers, (8)=(7), and solving for the evolution of the relative shadow price q_h/q_f by the two no-arbitrage conditions. Log differentiating (6), (7), (8) and combining these equations in levels with (9) yield after some manipulation the last dynamic equation.

6.2 Calibration

The real-life data we want to explain are the growth rate of GDP and the relative wage in the early 80s for the US. GDP growth for the US is close to 3% annually. In the simulation we use 3%. The relative wage of unskilled (high school or less education) versus skilled (college educated) workers is approximately 0.73 in 1979 (see Davis, 1993).

The following parameters are used in the simulations. OECD (1993) reports that 17% of the population is college educated and hence 83% has less education in 1980. Hence we use a ratio for L_N/H_N of 5. A common value for the rate of time preference is 0.05. The inverse of the intertemporal rate of substitution σ is taken to equal 1, to limit the number of cases to be considered. For *z*, the level of the GPT, we use 0.1 and 1. The productivity of assimilation is set to one. 1-α indicates the importance of the past experience in own productive knowledge accumulation and indicates the degree of decreasing returns to skilled labour in knowledge production. For 1-α estimations are available varying from 0.17 to 0.38.[30] For α we take 0.5, 0.6 and 0.7. Finally ϕ and the productivity of research (A_h) remain to fit the model to generate the desired growth rate and relative wage.

Table 6.1 Calibration

		α					
		0.5		0.6		0.7	
z	1	ϕ=3.76	A_h=0.082	ϕ=1.05	A_h=0.035	ϕ=0.34	A_h=5e-4
	1	ϕ=3.76	A_h=0.026	ϕ=1.05	A_h=0.009	ϕ=0.34	A_h=0.001

The other parameter values are: H_N=1, L_N=5, A_f=1, σ=1 and θ=0.05

Table 6.1 shows that depending on the size of the decreasing returns in research, a

[30] Dinopoulos and Thompson (1995, 1996).

fishing out results emerges (ϕ>1) if α is relatively small. With higher α's a lower level of ϕ is able to replicate the data.

For the steady states in the shaded areas in Table 6.1, hence for a ϕ larger than and less than unity, we will analyse the dynamics in the next section.

6.3 Dynamic analysis of a GPT with an application to the wage-inequality debate

The purpose of this section is twofold. First, we want to show that the model is able to generate a slowdown in the growth rate following the arrival of a new GPT. Second, we confront the predictions of the model with the data on the wage-inequality debate. Numerical analyses are carried out with a shooting routine. As the model has one state variable and two jump variables the model is globally saddle-path stable with one negative and two positive roots. All numerical examples considered fulfill this requirement.[31]

Case (i) A new GPT

In Case (i) Q and h enter additively into the knowledge pool, see equation (4A). In this case, the economy returns to the original steady state (see section 5). Figure 3 panel b shows the growth rate after the arrival of a new GPT.[32] The arrival of a new GPT enlarges the knowledge pool (the dashed line in panel a) and makes it relatively attractive to invest in assimilation.[33] Hence less is invested in reseach to accumulate directly productive knowledge and at the arrival of a new GPT the growth rate jumps down.[34] The assimilation effort lowers R and hence increases the effectiveness of reseach. So, after the above-steady-state effort in assimilation, research intensity is increased and hence growth rises and the assimilation of the GPT pays off. In the long run, the GPT has a negligible impact and the economy returns to the original (steady

[31]In case (i) a second state variable prevails, see Appendix D for computational details. In this case the numerical examples have two positive and two negative roots.

[32]The simulation shown in Figure 3 is based on the calibration exercise α=0.7 and z=1 as predetermined parameters.

[33]Note that in the transition R is not an appropriate indicator of the knowledge gap.

[34]If GDP also registered intangible investments the slowdown in growth would not occur.

state) growth rate.[35]

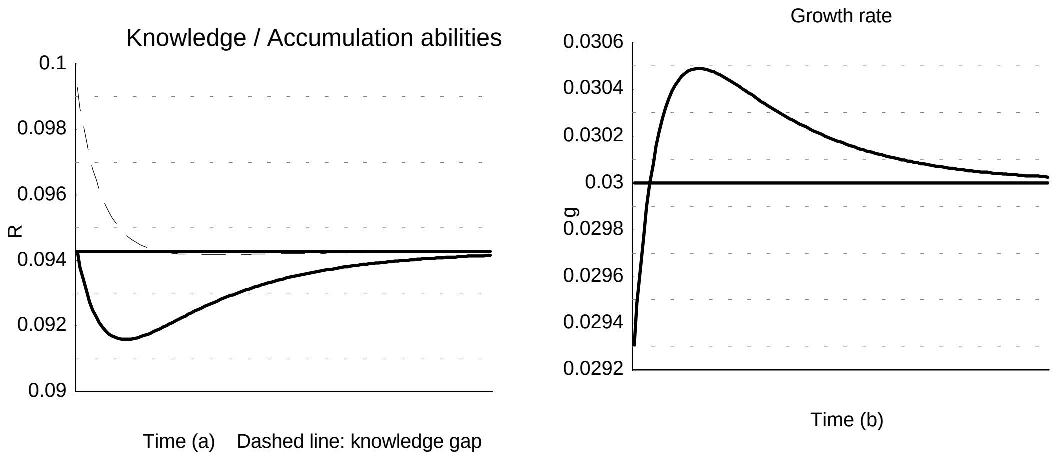

Figure 3 Dynamics of a GPT in Case (i)

Case (ii) An ideas generating GPT

In this case, *h* and *Q* enter multiplicative in the knowledge pool, (curve (4M)). An unanticipated new GPT arrives at time 0 when the economy is in the steady state (the steady state with a α=0.6, ϕ = 1.05 and z=0.1). The level of the GPT is increased by 15% from 0.1 to 0.115. In Figure 4 the horizontal lines indicate the initial steady state whereas the curved lines indicate the transition to a the new steady state. It is easily seen that a more sophisticated GPT implies a lower equilibrium knowledge gap, a higher steady state growth rate and a lower relative wage. On impact, the state variable *R* is, by definition, not affected. The new GPT, however, enlarges the knowledge pool and makes it very attractive to invest in assimilation, therefore *u* jumps down (panel b). That is, the allocation of skilled labour immediately jumps towards assimilation activities at the "cost" of accumulating productive knowledge. Therefore with a given ratio of productive knowledge to accumulation capabilities less research activity implies a lower growth rate of productivity in final goods production. Therefore the growth rate of GDP is lower initially (panel c).

Figure 5 presents the results of an equivalent shock given to the steady state Table 6.1 with the same *z* and α=0.7 (here ϕ<1). The qualitative dynamics depicted are

[35]At impact the relative wage of unskilled workers jumps down (not shown).

similar to the dynamics depicted in Figure 4 as far as panel (a), (b) and (c) are concerned. A discussion of panel (d) is postponed.

Summarizing the results so far we can conclude the following. Independent of the question whether the emergence of a new GPT will alter the long run growth rate the emergence a widely applicable technology as such will lead to stagnating productivity growth at impact.

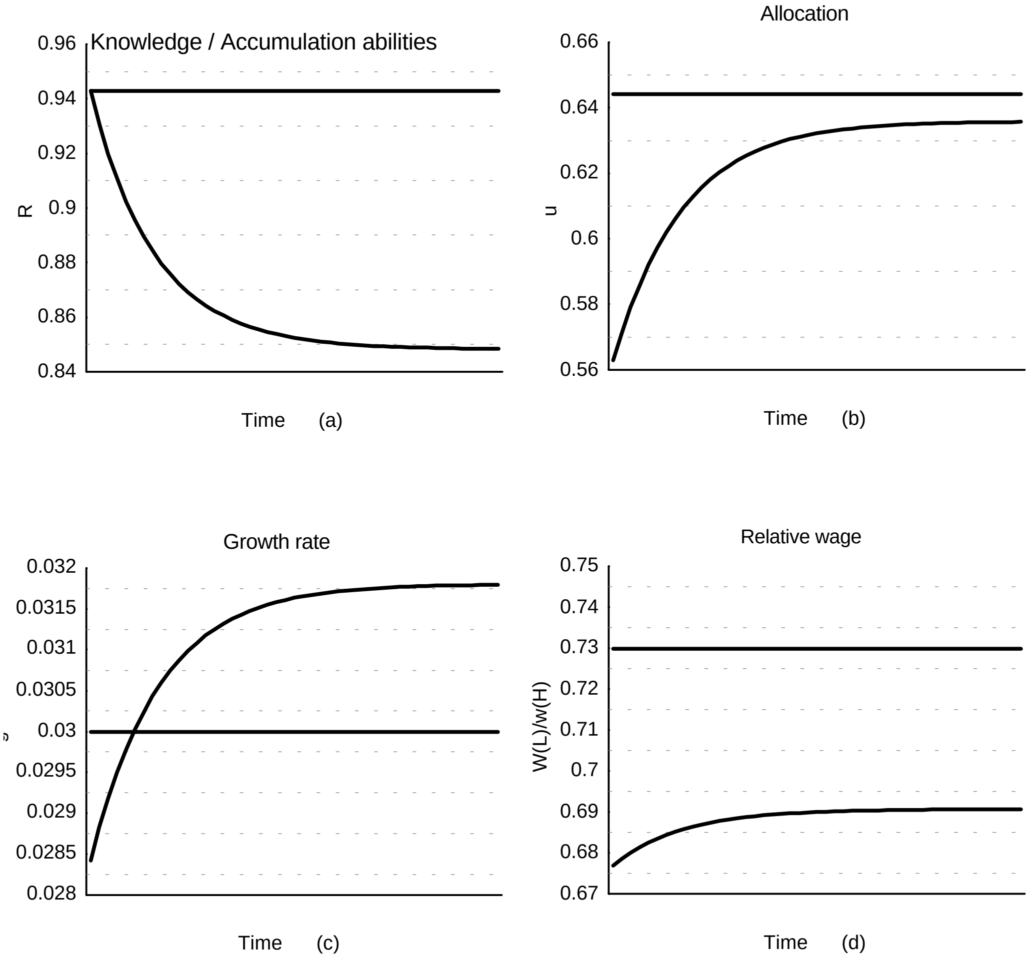

Figure 4. Dynamics of a GPT I in Case (ii), $\phi > 1$

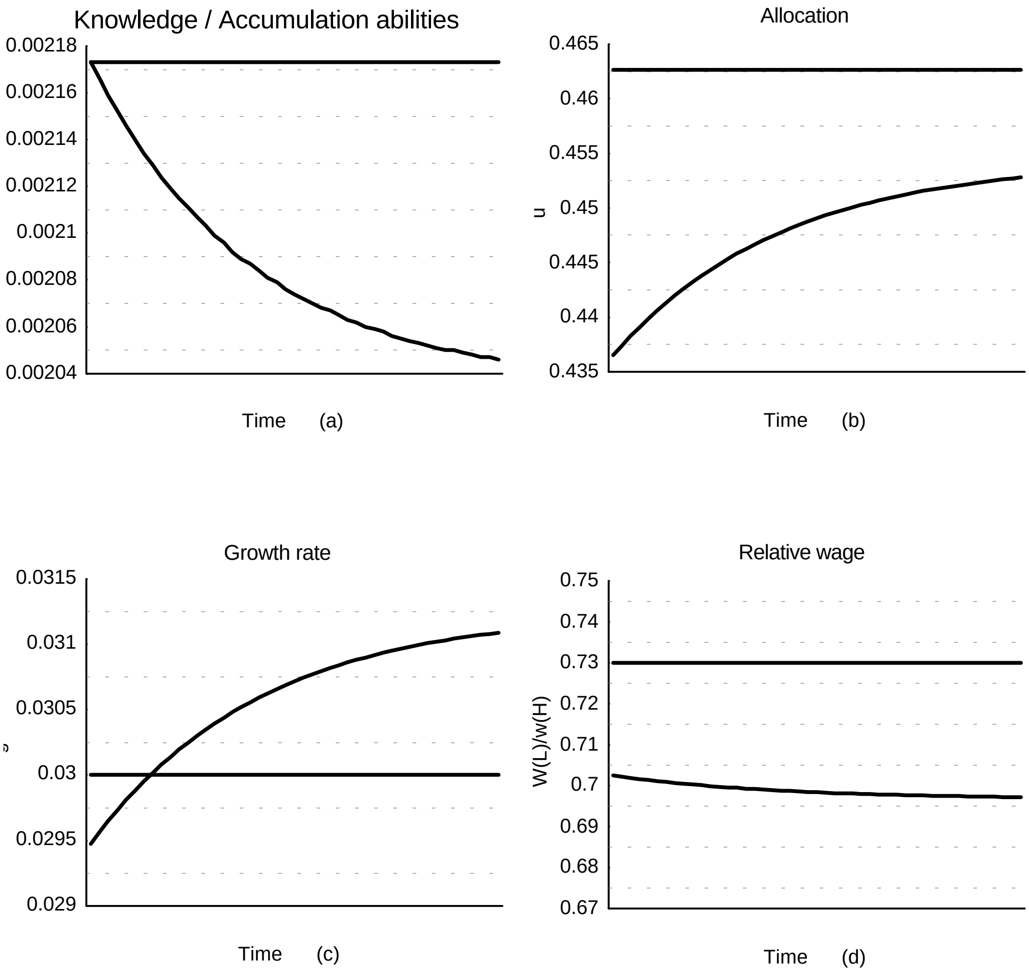

Figure 5 Dynamics of a GPT II in Case (ii), ϕ<1

How do the results match with the discussion on wage inequality?

One lesson that is immediately learned is that the question posed by Krugman (1995):"has the growth in total factor productivity been sufficient to be consistent with the large changes we have actually seen in factor prices?" (p. 7, 8) can be answered positively. The lower growth rate is even necessary to increase wage inequality. A point that should be made before discussing the actual results is that empirical research is required to determine the magnitude of the shock to the GPT that we have modelled here. For now such information is lacking. Comparing Figure 4 to Figure 5 shows that the qualitative dynamics differ between the two sets of parameters we have used. In the simulation with a $\phi>1$, Figure 4, the bad news for unskilled workers is over: at impact the relative wage jumps down whereas during the transition the relative position of unskilled workers improves. In the simulation reported in Figure 5 the relative wage of unskilled workers increases further. The intuition for the difference is that the return to assimilation declines quickly due to assimilation if $\phi>1$ (see (10)). As can be seen in , Figure 4, u increases much faster than in the case where ϕ is small, Figure 5.

How do our results match quantitatively? Davis (1993) reports a ratio of unskilled relative to skilled worker wages of 0.66 for 1987 (compared to 0.73 in 1979). The simulations yield a wage ratio that is close to this number. The results are especially important in the light of the puzzle raised by Feenstra and Hanson "the large increase in the non-production wage share over the period 1979-1987 is primarily the result of an increase in the relative employment of non-production workers that *occurred in just two years*, 1979 and 1980." (italics added, 1996 p.8).[36]

[36]Of course in our model there is no reallocation of workers between production and non-production (reseach and assimilation) work but if the model allowed for substitution, the change in relative factor rewards would to some extent turn up in reallocation of workers.

7. Final remarks

Two empirical puzzles that emerged in the 80s are the increase in the skill premium and sluggish productivity growth despite a technological revolution. Previous literature deals with these empirical phenomena separately. This paper shows a natural way to integrate both. It has been shown that a superiour General Purpose Technology leads to a temporary slump in the growth rate and an increase in the skill premium at the same time.

Most importantly, this model overcomes the critique of earlier work on GPTs (especially HT's) that requires reallocation from research to production and vice versa. This model generates a cycle without reallocation of labour from production to research. Secondly, the model generates these cycles without necessarily assuming that the new GPT is incompatible with existing knowledge stocks.

To conclude, some avenues for future research will be discussed. The model has a separated labour market to stress that reallocation of labour from reseach to production is not necessary. However, in future work, the model could be extended to allow for substitution of labour between research and production to get a better grip on the implied magnitude and the driving forces of inequality. Together with substitution, decreasing returns to the combination of research and assimilation could be introduced. Long run growth is then driven by the emergence of GPTs. This could be endogenized also. Hence, if improvements of productivity with a given GPT are exhausted, the return to the development of a new GPT will increase. As a working hypothesis it is convenient to assume that a GPT arrives accidentally, it is however hard to imagine that the economic environment has no impact on its arrival rate at all.

References

Aghion, P., and P. Howitt, "On the Macroeconomic Effects of Major Technological Change," 1996, Discussion paper.

Andolfatto, D. and G.M. MacDonald, "Technology diffusion and Aggregate Dynamics," *Review of Economic Dynamics*, 1998, Vol.1, No. 2. 338-370.

Bartel, A.P. and F.R. Lichtenberg, "The Comparative Advantage of Educated Workers in Implementing New Technology," *The Review of Economics and Statistics*, LXIX, 1987, 1-11.

Bresnahan, T.F. and M. Trajtenberg, "General Purpose Technologies 'Engines of Growth'?," *Journal of Econometrics*, 1995, Vol. 65, 83-108.

Cheng, L.K. and E. Dinopoulos, "A Multisectoral General Equilibrium Model of Schumpeterian Growth," *Journal of Economic Dynamics and Control,* 1996, 20, 905-923.

David, P.A., "Computer and Dynamo. The Modern Productivity Paradox in a Not-Too Distant Mirror," mimeo, 1989.

David, P.A., "The Dynamo and the Computer: An Historical Perspective on the Modern Productivity Paradox," *AEA Papers and Proceedings,* 1990, 355-361.

David, P.A., "Computer and Dynamo. The Modern Productivity Paradox in a Not-Too Distant Mirror," in *Technology and Productivity: The Challenge for Economic Policy*, 1991, OECD, Paris.

Davis, S.J., "Cross-Country Patterns of Change in Relative Wages," in: O. Blanchard and S. Fischer (eds.) *NBER Macroeconomics Annuals 1992*, 1992, 239-300.

Dinopoulos, E. and P.Thompson, “Is Endogenous Growth Empirically Relevant,” University of Florida Working Paper No 95-96-8, 1995.

Dinopoulos, E. and P.Thompson, “A Contribution to the Empirics of Endogenous Growth,” *Eastern Economic Journal*; 1996, 22(4), 389-400.

Ericksson, C. and T. Lindh, “Growth Cycles with Technology Shifts and Externalities,” Uppsala University, Working Paper Series 1997:15, 1997.

Feenstra, R.C. and G.H. Hanson, “Globalisation, Outsourcing and Wage Inequality,” NBER Working Paper 5424, 1996.

Felli, L. and F. Ortalo-Magné, “Technological Innovations: Slumps and Booms,” CARESS Working Paper 97-17, 1997.

Greenwood, J. and M. Yorukoglu, “1974,” *Carnegie-Rochester Conference Series on Public Policy*, 1997, 46, 49-95.

Groot, H.L.F. de, and R. Nahuis, “Optimal Product Variety, Scale Effects, and Growth,” CentER Discussion Paper 9754, 1997.

Groot, H.L.F. de and R. Nahuis, “A Note on Spillovers and Welfare” Tilburg University, working paper, 1998.

Grossman, G.M., and E. Helpman, *Innovation and Growth in the Global Economy*, Cambridge, MA: MIT Press, 1991.

Freeman, R., “Are Your Wages Set in Beijing?,” *Journal of Economic Perspectives* Summer 1995, Vol. 9, Nr. 3, 15-32.

Helpman, E. and A. Rangel, “Adjusting to New Technology: Experience and Training,” NBER Working Paper No. 6551, 1998.

Helpman, E. and M. Trajtenberg, “A Time to Sow and a Time to Reap: Growth Based on General Purpose Technologies,” NBER Working Paper No. 4854, 1994.

Helpman, E. and M. Trajtenberg, “Diffusion of General Purpose Technologies,” NBER Working Paper No. 5773, 1996.

Jones C.I., “R&D-Based Models of Economic Growth”, *Journal of Political Economy*, 1995, 103, 759-784.

Jovanovic, B. and R. Rob, “Long Waves and Short Waves: Growth Through Intensive and Extensive Search,” *Econometrica*, 1990, Vol. 58, No. 6, 1391-1409.

Klundert, Th. van de and S. Smulders, “Growth, Competition and Welfare,” *Scandinavian Journal of Economics*, 1997, Vol. 99, No.1, 99-118.

Krueger, A.B., “How Computers Have Changed the Wage Stucture: Evidence from Microdata, 1984-89,”*Quarterly Journal of Economics*, 1993, 33-60.

Krugman, P.R., "Technology, Trade, and Factor Prices," NBER Working Paper No. 5355, 1995.

Nahuis, R. “On Globalisation, Trade and Wages,” FEW Research Memorandum 747 May 1997.

OECD, *Employment Outlook 1993*, 1993.

Peretto, P. and S. Smulders, “Specialization, Knowledge Dilution, and Scale Effects in an IO-Based Growth Model,” CentER Discussion Paper No. 9802, 1998.

Rustichini, A. and J.A. Schmitz, “Research and Imitation in Long-run Growth,” *Journal of Monetary Economics*, 1993, 27, 271-292.

Stein, J. C., “Waves of Creative Destruction: Firm-Specific Learning-by-doing and the Dynamics of Innovation,” *Review of Economic Studies*, 1997, Vol. 64, 265-288.

Stiglitz, J.E., “Learning to Learn, Localized Learning and Technological Progress,” In: P. Dasgupta and J. Stiglitz (eds.): *Economic Policy and Technological Performance*, 1987, Cambridge: Cambridge University Press.

Smulders, S. *Growth, Market Structure and the Environment*, Phd. Thesis, Tilburg University, Hilvarenbeek, 1994.

Yorokoglu, M., “The Information Technology Productivity Paradox,” *Review of Economic Dynamics*, 1998, Vol. 1, No. 2, 551-592.

Appendices

A. The linearization procedure

The linearization procedure of the model is somewhat complicated by the fact that some variables are stationary and others are non-stationary. This appendix explains the applied procedure (and draws heavily on Smulders, 1994).

The linearization procedure for static equations is standard. By taking total differentials and dividing by the initial value of a variable we obtain percentage deviations from the original steady state. To simplify notation we define variables with a tilde as :

$$\tilde{w} \equiv d\frac{w}{w} \tag{A.1}$$

A simple example is the mark-up relation,

$$\tilde{p} = \tilde{w}_l - \tilde{h} = 0 \tag{A.2}$$

as h is non-stationary the level is not determined. However, as the growth rate is stationary, the change in de growth rate can be determined. So if $\dot{x}/x = g_x$ the growth rate of a non-stationary variable reads in linearised form:

$$d(\dot{x}/x) = g\tilde{g}_x \qquad \frac{d(\dot{x}/x)}{\dot{x}/x} = \tilde{g}_x \quad . \tag{A.3}$$

Growth rates of stationary variables (examples are q_h, q_f but also u; required for the log-time differentiated version of the optimality condition (7)) can be derived as follows:

$$d(\dot{y}/y) = \frac{1}{y}d\dot{y} - \frac{\dot{y}}{y^2}dy = \frac{1}{y}d\dot{y} \quad , \tag{A.4}$$

where we use the fact that as we linearize a stationary variable around the steady state these variables grow at rate zero. By definition:

$$d\dot{y} \equiv \dot{y}^{\,new} - \dot{y}^{\,old} \quad , \tag{A.5}$$

and $\dot{y}^{old} \equiv 0$. To derive $\dot{y}^{new}/y$, the following steps are helpful:

$$dy \equiv y^{new} - y^{old} \quad , \tag{A.6}$$

is equivalent to:

$$y^{new}=y^{old}\left(1+\frac{dy}{y^{old}}\right) \quad . \tag{A.7}$$

Differentiating with respect to time yields

$$\dot{y}^{new}=\dot{y}^{old}\tilde{y}+\dot{\tilde{y}}y^{old} \quad . \tag{A.8}$$

And use again that $\dot{y}^{old}$ is 0 by definition; hence we derive:

$$\frac{\dot{y}^{new}}{y^{old}}=\dot{\tilde{y}} \quad . \tag{A.9}$$

Note that in the new steady state $\dot{\tilde{y}}=0$.

Finally, z is the ratio of two non-stationary variables. As all non-stationary variables grow at a common rate this ratio itself is a stationary variable. Define $z=x_1/x_2$. The rate of change of the stationary variable is difference between two growth rates of the non-stationary variables:

$$d(\dot{z}/z)=d\left(\frac{\dot{x_1/x_2}}{x_1/x_2}\right)=g\tilde{g}_{x_1}-g\tilde{g}_{x_2} \quad . \tag{A.10}$$

In the steady state all non-stationary variables grow at a common constant rate hence $\dot{\tilde{z}}=0$.

B. Steady state analysis

B.1 Steady state ratios

To solve for the asset portfolio in the steady state use (7) and (8) and substitute the two accumulation equations(2),(3):

$$\frac{q_h h}{q_f f}=\frac{u}{1-u}\frac{1}{1-\alpha} \quad . \tag{B.1}$$

From (10) is alternatively solved for the asset portfolio by substituting (2) and (3), hence:

$$\frac{q_h h}{q_f f}=\frac{1}{\alpha}\left((\frac{r}{g}-1)+\phi\right) \quad . \tag{B.2}$$

The transversality condition learns that *r/g>1*. Combining the previous two equations yields:

$$\frac{u}{1-u}=\frac{1-\alpha}{\alpha}\left((\frac{r}{g}-1)+\phi\right) \quad . \tag{B.3}$$

Where the LHS is the ratio of the wage bill of workers engaged in research versus those in assimilation. It is convenient for later reference to derive the following expression from the no-arbitrage condition for research (9) (use (7)):

$$\omega\frac{L_N}{H_N}\frac{1}{u}=\frac{1}{1-\alpha}(\frac{r}{g}-1+\alpha) \quad . \tag{B.4}$$

The LHS is the ratio of the wage bill of unskilled workers versus workers engaged in research.

B.2 Linearization around the steady state

Linearization around the steady state the ramsey rule yields:

$$\tilde{g}_x=\frac{r}{\sigma g}\tilde{r}-\frac{\theta}{\sigma g}\tilde{\theta} \quad . \tag{B.5}$$

Equation (6) yields:

$$\tilde{w}_L=\tilde{h}, \qquad \tilde{g}_{w_l}=\tilde{g}_h \quad . \tag{B.6}$$

The second expression is derived, as both variables are non-stationary (and divide both sides by g). Equation (7) yields:

$$\tilde{q}_h+\tilde{A}_h-\alpha\tilde{u}-\alpha\tilde{H}_N+(1-\alpha)\tilde{h}+\alpha\tilde{f}=\tilde{w}_H, \qquad \dot{\tilde{q}}_h-\alpha\dot{\tilde{u}}-\alpha g\tilde{g}_h+\alpha g\tilde{g}_f=g\tilde{g}_{w_h}-g\tilde{g}_h \quad . \tag{B.7}$$

Equation (8) yields:

$$\tilde{q}_f+\phi\tilde{z}+\phi\tilde{h}+(1-\phi)\tilde{f}=\tilde{w}_H, \qquad \dot{\tilde{q}}_f+\phi g\tilde{g}_h-\phi g\tilde{g}_f=g\tilde{g}_{w_h}-g\tilde{g}_f \quad . \tag{B.8}$$

The no arbitrage condition for knowledge accumulation, (9) and substituting (7) in there and linearizing yields:

$$\dot{\tilde{q}}_h=r\tilde{r}-r\left[\tilde{A}_h-\alpha(\tilde{h}-\tilde{f})+\frac{(1-\alpha)uH_N-\alpha\omega L_N}{\omega L_N+uH_N}(\tilde{u}+\tilde{H}_N)+\frac{\omega L_N}{\omega L_N+uH_N}(\tilde{L}_N+\tilde{w}_L+\tilde{w}_H)\right] \quad , \tag{B.9}$$

where the weights can be simplified by dividing numerator and denominator by uH_N and substituting for (B.4). This yields:

$$\dot{\tilde{q}}_h=r\tilde{r}-r\left[\tilde{A}_h-\alpha(\tilde{h}-\tilde{f})+((1-\alpha)\frac{g}{r}-\alpha)(\tilde{u}+\tilde{H}_N)+(1-(1-\alpha)\frac{g}{r})(\tilde{L}_N+\tilde{w}_L+\tilde{w}_H)\right] \quad . \tag{B.10}$$

The no-arbitrage condition for assimilation (10) can be rewritten (use (7) and (8)) as:

$$A_h(zh)^{\phi}f^{-\phi}H_N\left(\frac{\alpha}{1-\alpha}u+(1-\phi)(1-u)\right)+\frac{\dot{q}_f}{q_f}=r \quad . \tag{B.11}$$

Linearizing yields:

$$\dot{\tilde{q}}=r\tilde{r}-r\left[\tilde{A}_f+\phi\tilde{z}+\phi(\tilde{h}-\tilde{f})+\frac{\frac{\alpha}{1-\alpha}u-(1-\phi)u}{\frac{\alpha}{1-\alpha}u+(1-\phi)(1-u)}\tilde{u}+\tilde{H}_N\right] \quad . \tag{B.12}$$

The weight is again expressed in terms of r and g by substituting (B.3):

$$\dot{\tilde{q}}_f=r\tilde{r}-r\left[\tilde{A}_f+\phi\tilde{z}+\phi(\tilde{h}-\tilde{f})+(1-\frac{g}{r}(1-\phi))\left(1-\frac{(1-\phi)(1-\alpha)}{\alpha}\right)\tilde{u}+\tilde{H}_N\right] \quad . \tag{B.13}$$

And finally the two accumulation equations (2), (3) yield:

$$\tilde{g}_h=\tilde{A}_h+(1-\alpha)\tilde{u}+(1-\alpha)\tilde{H}_N-\alpha(\tilde{h}-\tilde{f}) \quad , \tag{B.14}$$

where (B.3) is substituted.

$$\tilde{g}_f = \tilde{A}_f + \phi\tilde{z} + \phi(\tilde{h} - \tilde{f}) - \frac{1-\alpha}{\alpha}\left(\frac{r}{g} - (1-\phi)\right)\tilde{u} + \tilde{H}_N \quad . \tag{B.15}$$

The steady state definition in (19) and (20) reads in linearized form:

$$\tilde{g}_x = \tilde{g}_h = \tilde{g}_f = \tilde{g}_{w_l} = \tilde{g}_{w_h} \equiv \tilde{g}, \qquad \dot{\tilde{q}}_h = \dot{\tilde{q}}_f = 0 \quad . \tag{B.16}$$

Finally the constant allocation in the steady state implies:

$$\dot{\tilde{u}} = 0 \quad . \tag{B.17}$$

B.3 An improved GPT: an example

To find the impact of an increased quality of the GPT on the steady state substitute the steady-state definitions (B.16) and (B.17) in equations (B.5)-(B.15). And set:

$$\tilde{\theta} = \tilde{A}_h = \tilde{A}_f = \tilde{H}_N = \tilde{L}_N = 0 \quad . \tag{B.18}$$

To solve for $\tilde{g}$ as a function of $\tilde{z}$ substitute (B.5) in (B.13) and plug in *($\tilde{h}$-$\tilde{f}$)*. *($\tilde{h}$-$\tilde{f}$)* follows from (B.15)=(B.15). This yields $\tilde{g}$ is a function of $\tilde{u}$. Plug . *($\tilde{h}$-$\tilde{f}$)* in (B.15) to get a second expression for $\tilde{g}$ in terms of $\tilde{u}$. Solving for $\tilde{g}$ and simplifying yields Ξ in (23).

B.4 Comparative Statics

The parameters capturing the efficiency of the growth engines both have a positive impact on the growth rate, although the mechanism differs. An increase in the efficiency parameter of assimilation, A_f, increases the return to assimilation and hence induces a reallocation of skilled workers from research to assimilation. Obviously, the knowledge gap decreases as the efficiency of, and resources allocated to, assimilation increase. An increased efficiency of research, A_h, leads *cet. par.* to an increased return to research and an increase in the growth rate of productive knowledge. This would induce reallocation of workers towards research, where account has to be taken of the fact that decreasing returns to research labour mitigates this effect. Productive knowledge spills over to the knowledge pool and hence increases the return to assimilation. This induces reallocation towards assimilation. On balance more resources are allocated to assimilation, but the knowledge gap decreases. Obviously, marginal increases in the productivity of skilled workers lead to increased inequality.

Table B.1 Comparative statics

	θ	A_f	A_h	H_N	L_N
g	'-'	'+'	'+'	'+'	'0'
w_l/w_h	'+'	'-'	'-'	'?'	'-'
R	'+'	'-'	'+'	'-'	'0'
u	'+'	'-'	'-'	'-'	'0'

Increasing the number of skilled workers increases the rate of growth. This result is the scale effect that prevails in many growth models. A larger economy, what should be interpreted as more skilled workers per firm, generates a higher growth rate. An increase in the amount of skilled workers leads to a decrease in the share of workers doing research, *u,* due to decreasing returns in research. As a larger chunk of the skilled workforce assimilates, the knowledge gap decreases. At first sight counter intuitively, an increase in the amount of skilled workers does not necessarily imply an increase in the relative wage of unskilled workers. An increase in the number of skilled workers makes them relatively abundant, and keeping everything else constant this would cause downward pressure on wages of skilled worker, but this need not be so. Keep in mind that skilled workers are only active in research and assimiliation. So, to get a decline in the relative wage of unskilled workers the growth engine should become marginally more efficient as more workers are employed. Necessary for this odd result to hold is that $\phi < 1$, hence the case where increased accumulation capabilities ease future assimilation (if ϕ is high the best ideas are fished out first and increased assimilation activity is not a very efficient activity).[37] The intuition for the odd result goes as follows. The share of workers in assimilation increases[38] and with very low ϕ the activity remains very efficient and does not rely very much on the general knowledge pool. Hence, the role for spillovers is small. The latter can also be understood from the other side: the fact that relatively less spillovers are generated due to the reallocation of research towards assimilation implies that spillovers better should not be important, otherwise the growth engine could not be more efficient.[39]

[37]The exact condition is not very informative.

[38]From equation (22) in the main text is seen that the relative wage of unskilled workers is increasing in *u*.

[39]The role for α in the condition is less clear, as an increase in α implies stronger diminishing returns to skilled workers in research at a point in time but also increases the share of accumulation capabilities in the research engine.

C. Corner Solutions

C.1 Binding inequality constraints.

In the assimilation technology (3) there are no Inada conditions, therefore at a certain rate of imbalance ($h/f \neq (h/f)^{\text{steady state}}$) one of the inequality constraints in Table 3.1 will become binding. Suppose the ratio of productive knowledge to accumulation capabilities is too low for further investment in accumulation capabilities to be attractive. Hence the condition $H_A \geq 0$ is binding.

The firm problem then becomes:

$$\max \ V_i = \int_0^{\infty} e^{-r_t} \left(x_{it} p_{it} - H_{Rit} w_{Ht} - L_{it} w_{Lt} \right) dt \quad , \tag{C.1}$$

subject to: (1), (2), and (15). This yields (6),(7), and (9). The labour market condition for skilled workers now becomes

$$\frac{H}{N} = H_{Ri} \equiv H_N \quad . \tag{C.2}$$

Now the growth rate is easily determined as:

$$\hat{h} = A_h f^{\alpha} (H_N)^{1-\alpha} h^{-\alpha} \quad . \tag{C.3}$$

Having an initial value for h_0 , the growth rate is determined, recall that f is constant. The relative change in the growth rate is $-\alpha\hat{h}$. Hence, the rate of growth is increasing in the imbalance and decreases after the shock. Use (7) and take w_H as a numeraire and log-differentiate to see:

$$\hat{q}_h = -(1-\alpha)\hat{h} \quad . \tag{C.4}$$

Hence, the shadow price of productive knowledge capital declines with the growth of productive knowledge. At a certain h/f ratio the inequality constraint ceases to be binding and the economy enters the regime discussed in the main text.

C.2 λ<1 and Z homogenous of degree one in h.

The approach proposed by Jones (1995) yields a model with undesirable features given the segmented labour market. Insert the definition of u in (2) and (3), devide respectively by h and f and log-differentiate to time, to get:

$$\hat{g}_h = (1-\alpha)\hat{u} + (1-\alpha)\hat{H} - \alpha\hat{h} + \alpha\hat{f} \quad , \tag{C.5}$$

$$\hat{g}_f = \phi\lambda\hat{Z} + (\lambda - 1 - \phi\lambda)\hat{f} - \frac{u}{1-u}\hat{u} + \hat{h} \quad . \tag{C.6}$$

To have a steady state with constant growth rates and allocation requires:

$$\hat{h} = \hat{f} \quad \wedge \quad (\lambda\phi + 1 - \lambda)\hat{f} = \lambda\phi\hat{Z} \quad , \tag{C.7}$$

which implies:

$$\hat{Z} = \frac{\lambda\phi - \lambda + 1}{\lambda\phi}\hat{f} \quad \vee \quad \hat{f} = \hat{h} = 0 \quad . \tag{C.8}$$

In the main text (Section 4.1) we ruled out the first equality, as the parameter combination on the RHS exceeds one. The second equality is hence needed to have a steady state. Hence a steady state with growth rate of zero would result. The allocation of skilled workers in longer determined in that case.

Suppose skilled workers could be allocated symmetrically to firms to do R&D, a positive growth rate could be generated by a positive rate of growth in the endowment of skilled workers. No steady allocation could be reached as u is affected by changes in H, see Table 4.1 in the main text.

D. Dynamics of GPT with the (4A) curve

The dynamic analysis of a GPT that enters the knowledge pool additively complitates the derivation of the differential equations describing the economy's behaviour outside the steady state somewhat. Taking the (4A) curve, we need to follow the same procedure as in section 6.1. Taking into account that logaritmic differentiation to time of Z yields:

$$\hat{Z} = \frac{\dot{h}}{h+Q} = \frac{A_h R^{-\alpha}(uH)^{1-\alpha}}{1 + \frac{Q}{f}\frac{1}{R}} \tag{D.1}$$

where the last equality follows from (2) and the definition of R. Using this and the fact that $Z = h + Q$ the differential equation for u can be written as:

$$\begin{aligned} \dot{u} = \; & \frac{u}{\alpha}\left(\left(\frac{\alpha}{1-\alpha}u + \alpha(1-u) \right) A_f H_N \left(R + \frac{Q}{f} \right)^{\phi} - (1-\alpha)\omega L_N A_h (uH_N)^{-\alpha} R^{-\alpha} \right) + \\ & \frac{u}{\alpha}\left(-\phi \left(\frac{A_h (uH_N)^{1-\alpha} R^{-\alpha}}{1 + \frac{Q}{f}\frac{1}{R}} \right) \right) \quad , \end{aligned} \tag{D.2}$$

The equation for the ratio of productive knowledge to accumulation capabilities now looks like:

$$\dot{R}=R\left(A_h(uH_N)^{1-\alpha}R^{-\alpha}-A_f\left(R+\frac{Q}{f}\right)^{\phi}(1-u)H_n\right) \quad . \tag{D.3}$$

The dynamics of relative wages are determined by:

$$\dot{\omega}=\omega\left((1-\phi-\sigma)\left(\frac{A_h(uH_n)^{1-\alpha}R^{-\alpha}}{1+\frac{Q}{f}\frac{1}{R}}\right)-\theta+\frac{\alpha}{1-\alpha}(uH_N)A_f\left(R+\frac{Q}{f}\right)^{\phi}\right) \quad . \tag{D.4}$$

Finally the differential equation for Q/f is:

$$\frac{\dot{Q}}{f}=-\frac{Q}{f}A_f\left(R+\frac{Q}{f}\right)^{\phi}(1-u)H \quad . \tag{D.5}$$

Note that f grows over time and Q is constant, hence that in the limit the system of differential equations evolves to the one in the main text, as the last differential equation tends to 0=0.